The Importance of the Empty Set and Noncommutative Geometry in Underpinning the Foundations of Quantum Physics

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Abstract

Unlike relativity quantum mechanics was not developed originally as a spacetime theory. Noncommutative geometry radically changed the situation. The present work builds upon noncommutative geometry and Cantorian spacetime theories to give a mathematical explanation coupled to a general and obvious physical reasoning for the quantum contra-intuitive results associated with particle wave duality, state vector reduction and related aspects at measurement. In particular we stipulate that a quantum particle is represented by a Cantor point via the dimensions of a Cantorian zero set given by dim (P) = $(0,\phi)$ while the quantum wave is represented by the emptiness between disjointed Cantorian points, i.e. a complementary empty Cantorian set of dim (W) = $(-1,\phi)$ where $\phi = (\sqrt{5}-1)/2$. Measurement interferes with the empty set and renders it non-empty and that accounts in various ways for the wave collapse and the disappearance of the interference fringes.

Keywords: Discontinuous spacetime , noncommutative geometry, fractal , particle wave duality, golden ratio

1. Introduction

The Authors admit to having always been fascinated by the works on noncommutative geometry [1] and on E-infinity theory [2,3]. In particular it did not go unnoticed that one of the most distinguished mathematicians of our time, who developed noncommutative geometry, does not use mathematics in the way it is usually used by almost everyone, namely as a utility to model physics and not much more than that. It seems to us that Connes feels that the ultimate reality is not physical but rather mathematical [1]. Being a very deep mathematician made him understand fundamental mathematics as if it were physics and this way he discovered the real meaning of fractals and the golden mean which he summed up in his dimensional function inspired by the work of von Neumann on continuous geometry [1-3]

$$D = a + b\phi, \tag{1}$$

where $a, b \in \mathbb{Z}$ and ϕ is the golden ratio[4], $\phi = (\sqrt{5} - 1)/2$.

In turn and due to his deep understanding for the work of Penrose on fractal tiling [1], he was able to discover the physical meaning to his function. On the other hand El Naschie seems to proceed from the opposite view point or maybe one should say from a complementary direction. In E-infinity [2,3] he elevates physical observation to almost the status of mathematical theorems. He is not a mathematician nor in fact a physicist, being an engineering scientist by training. However his knowledge of nonlinear dynamics enables him to feel his way, finding the right mathematics for what we perceive as physics. In addition El Naschie was familiar with Penrose fractal tiling, random Cantor sets and the golden mean[4] from his work on deterministic chaos, the KAM theorem and fluid turbulence in engineering. In this unconventional way of doing fundamental theoretical physics, he discovered his bijection formula [2,3] which is congruent to the dimensional function of von Neumann's continuous geometry and Connes' noncommutative geometry [1]

$$d_c^{(n)} = (1/d_c^{(0)})^{n-1} \tag{2}$$

and the golden mean theorem which states that

$$d_c^{(n)} = \langle d_c \rangle \cong \langle n \rangle \tag{3}$$

if and only if $d_c^{(0)} = (\sqrt{5} - 1)/2$ and $n = 4$. Here n is the topological Menger-Urysohn spacetime dimension and $d_c^{(n)}$ is the corresponding Hausdorff dimension. It is easily demonstrated that this formulation reproduces Connes' dimensional function [1] and a few more things and leads essentially to the same sweeping conclusions about the fractal reality of our noncommutative geometry, i.e. quantum spacetime and the meaning and reasons for many experimental results in quantum mechanics which are commonly perceived as paradoxical [5-8].

The authors worked extensively for a decade or so to show the role played by complexity theory in quantum physics [9,10]. They gave explicit and simple geometrical interpretations to the Hilbert cube [11,12] and its relation to the exceptional Lie groups, particularly E8 [2]. They further pointed out following the originator of E-infinity theory the vital importance of the Menger-Urysohn theory of transfinite dimension [13] and the physical deficiency in any fundamental theory which does not include the empty set dim empty, $(-1, \phi^2)$. This set possesses a negative Menger-Urysohn dimension $d_{MU} = -1$ and a Hausdorff $d_M = \phi^2$ where

$\phi = (\sqrt{5} - 1)/2$ is the golden mean. In addition to that it seems that E-infinity and noncommutative geometry are two of the rare theories which provided a theoretical framework for a deeper understanding of the result of the COBE experiment. Finally it was the combined work of our group with those working worldwide on E-infinity theory [15,20,24] which led to a mathematically consistent explanation of the set of quantum particles with dimension $P_Q = (0, \phi)$ and the set of quantum wave with dimension $P_W = (-1, \phi^2)$ as well as the real meaning of the collapse and disappearance of the interference fringes in the two-slit experiment [14,15] and at the measurement. This was previously considered a largely metaphysical part of quantum mechanics with or without Böhm guidance [5-8]. Another important line of investigation using related methodology is that of L. Nottale [2,9,16-18]. In the final analysis it was the work on E-infinity which brought all these different approaches of G. Ord and L. Nottale together under the same roof and connected them not only to the work of Connes and Penrose, but to a certain extent to many presently mainstream physics theories like string theory and loop quantum gravity [16,17]. We dare say that in our opinion when researchers of strings and loops incorporate the empty and the totally empty set into their work, that is to say fractals in a fundamental way, it will immediately be noticed that the result will be almost identical to the work on noncommutative geometry and E-infinity theory.

There are several essential mathematical concepts and theorems which are needed for a quick proper understanding of the present work. It is helpful for readers who like the authors of the present paper were not very familiar with the mathematics of the infinite to know and appreciate the following subjects:

First one needs to understand the role of Bijection which is essential for a rigorous counting and constitutes the beginning of set theory and the work of G. Cantor [2,3].

Second we need to understand and appreciate the method of complete induction. It is due to Pascal but if one goes back in history one will find that the ancient Chinese may have been there first. Third one has to look at the work and philosophy of Henri Poincaré to deepen our understanding of the complete induction method. Fourth is to look at the science of converging and diverging series. Fifth it is recommended to consider the various famous logical paradoxes. For instance, Russell's self-referential statement, which will eventually lead to the appreciation that fractals are self-referential. Sixth study the axiom of choice and look at the Banach-Tarski theorem which was considered in cosmology [19]. Seventh is a careful study of Cantor's diagonal argument to show that there are at least two types of infinity, namely (1) countable and (2) non-countable infinity. Note that the second is larger than the first and that E-infinity has cardinality larger than the continuum. Eighth may be to consider the work of Alan Turing who invented the Turing computer which is the quintessence of a computer without hardware [6]. Ninth look at number theory and particularly the concept of Dedekindian Cut and related mathematical issues. Tenth need to read the works of K. Gödel (1930) and P. Cohen (1963) on the continuum hypothesis. Eleventh it is essential to look at the work of Emile Borel. Note that a Borel Set as discussed by Wheeler was the starting point of E-infinity mathematics [2,3]. Twelfth connect all the above to fractals and relate the Menger Sponge to the Hilbert Hotel. Thirteenth, in order to get to physics one should apply all that the one has learned from the above to the work of Max Planck and his constant as well as Heisenberg and his matrices in addition to Schrödinger and his waves. That way one can gain with ease in a short time a good understanding of E-infinity theory [2,3]. Alternatively careful reading of two convenient summaries of the theory and results of E-infinity theory are given in [9,16,27-29].

2. The Cantorian manifold

The present work mainly presents what we consider to be a rather convincing resolution of the which way information paradox of the two-slit experiment [14,15] based on random fractal sets and the extended Menger-Urysohn transfinite theory of dimension [13]. This paradox neatly encapsulates nearly all the basic contra-intuitive results of orthodox quantum mechanics such as wave collapse, nonlocality, entanglement and being at two locations at the very same time. As hinted at in the introduction, we use for each fractal set two component dimensions [18-22]. The first component is an invariant topological dimension and the second component is a corresponding Hausdorff-fractal dimension calculated using a continuous dimensional function [1]. Starting from A. Connes function, Eq.(1), of Penrose noncommutative tiling, we demonstrate the equivalence of this function to that of the E-infinity bijection formula, Eq.(2), where $d_c^{(0)} = \phi = (\sqrt{5} - 1)/2$ is the topological invariant of the related dimension group [9,22,28,29].

The golden ratio $\phi = (\sqrt{5} - 1)/2 = 0.618033989$ which was experimentally observed in quantum mechanics rather recently in the Helmholtz Centre-Germany, in quantum mechanics [23] arises here naturally from the requirement of a random fractal horizon of a noncommutative geometry [1-3] related to the compactified Klein modular curve [3,10]. This curve with 336 + 3 = 339 hierarchal degrees of freedom or isometries is also equivalent to the holographic boundary of E-infinity spacetime [3,9]. Since Penrose tiling is essentially a quotient space, the holographic boundary as well as the E8 exceptional Lie symmetry group may be regarded as a deformation of this tiling and the compactified SL (2,7) which is basic to E-infinity theory [9].

One of the most important steps taken in the present work is to follow a proposal [20] for the identification of the empty set with wave-like quantum probability and the true vacuum while quantum-like particles are described as a zero set. Consequently the two component dimension relevant to the particle is [18-22]:

$$\text{dim (particle set)} = P(d_{MU}, d_H) = P(0, \phi) \tag{4}$$

where d_{MU} is the Menger-Urysohn dimension and d_H is the corresponding Cantorian or Hausdorff dimension. For the quantum wave on the other hand we have [22]

$$\text{dim (wave set)} = W(d_{MU}, d_H) = W(-1, \phi^2)\ . \tag{5}$$

Since we have identified the quantum wave which is devoid of energy matter and momentum with the empty set, which contains no elements what so ever not even zeros, but is never the less structured and possesses a Hausdorff fractal dimension, it follows then as an almost trivial conclusion that any attempt to observe the two-slit experiment while in progress will render the empty set non-empty and instantly lead to the disappearance of the pattern caused by wave interference, leaving the fractal zero sets, i.e. particles form a patternless randomly distributed dots on the detection screen as the only observable. Using the preceding conclusion it is then a relatively straightforward mathematics which leads us to the form of semi-manifold which supports the preceding process. It turns out that this semi-manifold is a fractal quotient manifold of the Gaussian type [22] akin to the space of Penrose fractal tiling with a combined Hausdorff dimension given by [19,28,29]

$$D_H = \frac{\dim(particle) \oplus \dim(wave)}{\dim(particle) \otimes \dim(wave)} = \frac{(\phi) + (\phi^2)}{(\phi)(\phi^2)} = 4 + \phi^3 = 4 + \cfrac{1}{4 + \cfrac{1}{4 + \ldots}} = 4.236067977\ , \tag{6}$$

and a combined Menger-Urysohn dimension

$$D_M = \frac{0 + (-1)}{(0)(-1)} = \infty \tag{7}$$

as well as an average combined topological dimension equal to

$$< D > = \frac{(1/2) + (1/2)}{(1/2)(1/2)} = 4\ . \tag{8}$$

In other words this manifold is nothing else but the core of E-infinity Cantorian spacetime which may be envisaged as an infinite hierarchy of concentric four dimensional cubes resembling self-similar Russian dolls which is the basic picture of E-infinity spacetime. It is extremely interesting at this stage to note that although E-infinity spacetime is a discretum, i.e. totally disconnected point set semi-manifold, its cardinality is strictly larger than that of the continuum [1].

3. **The empty set – An elementary derivation of its topological dimension minus one (-1)**

What is the dimension of a 3D cube boundary? This is an utterly trivial question since it is clearly an area, i.e. a surface which is 2D. That means

$$3D(\text{cube}) - 1 = 2D(\text{Surface}).$$

Next we ask a second equally trivial question, namely what is the dimension of the boundary of a 2D surface? It is obviously a one dimensional line

$$2D(\text{surface}) - 1 = 1D(\text{line}).$$

Finally what is the dimension of the boundary of a line? This is evidently a zero dimensional point. That means

$$1D(\text{line}) - 1 = 0D(\text{point}).$$

It seems natural that by induction one could write a general expression for the above in the form

$$D(\text{boundary}) = n - 1$$

where n is the dimension of the geometrical object for which we would like to know the dimension of its boundary. This is a trivial application of the induction principle. However what if we want to extend this formula below a point just as we usually extended it above a 3D cube? We routinely deal in higher geometry with 4D and nD cubes as discussed by Coxeter and studied thoroughly in the context of E-infinity in Refs. [11,12]. In this case we use induction to say that the boundary of a point has a dimension [19-22]

$$D = D(0) - 1 = 0 - 1 = -1.$$

This is the dimension of the classical empty set as deduced for the first time by P. Urysohn and studied by K. Menger [10]. Unlike previous steps, this step is by no means a trivial or an obvious one and only looks deceptively simple. The stumbling block however is that we know of no "ordinary" geometrical object for which we can use the method of complete induction and find for instance

$$1 - \phi = \phi^2$$

and so on until we find D = $-\infty$ for which the Hausdorff dimension by bijection is equal to

$$d_c^{(-\infty)} = (1/\phi)^{-\infty-1} = 0$$

The only exceptional geometry for which the above is true is 'non-ordinary' Cantorian geometry [2,9]. In Cantor sets you can split a point in two by graphical computer zooming. It is the important discovery of the present paper that this transfinite set theoretical formulation can be used in quantum physics where the quantum particle is represented by the dimension doublet of the zero set $d_c^{(0)} = \phi$ namely

$$\dim P_Q \equiv (0 , \phi),$$

while the quantum or ghost wave is represented by the dimension doublet of the empty set

$$\dim W_Q \equiv (-1,\ \phi^2)$$

as we mentioned in the introduction. It is important to always keep in mind that the empty set is the 'surface', i.e. the neighbourhood of the zero set. Consequently the wave is in a sense the surface of the particle [18-22].

4. The relation between Connes' noncommutative geometry dimension function and E-infinity bijection formula

We start by the dimension function of the noncommutative quotient space representing the well known Penrose tiling [1],

$$D(a,b) = a + b\phi \ .$$

Our aim is to show that under certain conditions this dimension function will yield the bijection formula of E-infinity [9],

$$d_c^{(n)} = (1/\phi)^{n-1}$$

Let us set D_n (a, b) to be first $D_o \equiv D$ (0, 1) and $D_1 = D$ (1, 0). Subsequently we add a_i and b_i following the Fibonacci scheme as follows:

$$D_0\,(0, 1) = 0 + \phi = \phi$$
$$D_1\,(1, 0) = 1 + (0)\,\phi = 1$$
$$D_2\,(0 + 1\,,\ 1 + 0) = 1 + \phi = 1/\phi$$
$$D_3\,(1 + 1\,,\ 0 + 1) = 2 + \phi = (1/\phi)^2$$
$$D_4\,(1 + 2,\ 1 + 1) = 3 + 2\phi = (1/\phi)^3$$
$$D_5\,(2 + 3,\ 1 + 2) = 5 + 3\phi = (1/\phi)^4$$

and so on. By induction we conclude that

$$D_n = (1/\phi)^{n-1}$$

This is the bijection formula of E-infinity theory [2,3,9], as shown in Eq.(2), where $d_c^{(0)} = \phi$.
However we see that the bijection notation is more compact and economical and we know two dimensions at once; the n is the Menger-Urysohn dimension d_{MU} while $d_c^{(n)}$ is the Hausdorff dimension. Consequently it is easy to extend the formula to negative dimensions so that we would have for instance[18-22]

$$d_c^{(-1)} = (1/\phi)^{-1-1} = (1/\phi)^{-2} = \phi^2$$

which is our empty set dimension binary

$$\dim\,(\text{empty set}) = (-1\,,\ \phi^2)$$

where -1 is topologically invariant Menger-Urysohn dimension while ϕ^2 is the Hausdorff dimension which is not topologically invariant but extremely useful. We see clearly that the totally empty set, by complete induction, must be [18-22]

$$d_c^{(-\infty)} = (1/\phi)^{-\infty-1} = (1/\phi)^{-\infty} = \phi^{\infty} = 0.$$

The zero set on the other hand is [18-22]

$$d_c^{(0)} = (1/\phi)^{0-1} = (1/\phi)^{-1} = \phi$$

as is well known and in full agreement with the dimensional function of noncommutative geometry [1]. It is also clear that the zero set and the empty set are complementary because $1-\phi=\phi^2$.
We cannot stress enough that it is the most important conclusion of our theory that transfinite set theoretical formulation can be used in quantum physics where the quantum particle is represented by the dimension binary of the zero set $d_c^{(0)} = \phi$, namely [2,9,24]

$$\dim P_Q = (0, \phi)$$

while the quantum, wave reminiscent of the ghost or guiding wave of Einstein and Bohm is represented by the dimension binary of the empty set [18-22]

$$\dim W_Q = (-1, \phi^2)$$

as mentioned in the introduction as well as section 3. Note that the similarity with the guiding wave proposal is only partial and the two theories are by no means identical. For instance in the Einstein-Bohm ghost or guiding wave picture there is no wave collapse at all exactly as in the many world picture of Everett [8,15]. Note also that the Cantorian point representing the particle is a point as well as being an entire Cantor set when magnified. Thus it is a superposition of infinitely many points. The obvious reason for this contra-intuitive geometrical picture is the zero measure of the Cantor set.

Discussion and conclusion

In the words of Sir R. Penrose the two-slit experiment is the 'archetypical quantum-mechanical experiment'. The subject was examined with painstaking accuracy rather recently by Nobel laureate Sir A. Leggett [23,25]. The present work however takes new direction based on an important mathematical development in applying noncommutative geometry and higher dimensional transfinite set theory to the foundations of quantum mechanics [18-22]. Let us use the same terminology of Penrose regarding the U, O and R processes and results of measurement [14,26]. We recall that the empty set is de facto two identical things at the very same time, namely the surface or the topological neighbourhood of the zero set as well as being the guiding quantum wave. Similarly the zero set is a Cantorian fractal point as well as the quantum particle guided by the 'ghost' wave. Compared to the particle, the wave has very large numbers of degrees of freedom and consequently very large entropy. This may be understood in a very elementary manner by recalling that the wave is the surface of the particle and it is evident that the smaller the particle, say a sphere, the larger is the ratio between its surface area and its volume. When the volume becomes very small, the ratio becomes very large. This is well known from everyday experience with dissolving a cube of sugar or sugar grains in our tea and coffee. This is another intuitive way of looking at the disproportionally large difference in the degrees of freedom to the benefit of the wave vis-à-vis the particle aspect of the quantum. Now on taking measurement of this particle-wave packet we inevitably enter into the wave and consequently into the domain of the empty set. That way the empty set becomes non-empty and practically reduced or jumps to at best, a zero set. That is to say, only the particle with its minimalistic degrees of freedom corresponding to a much smaller entropy than the wave becomes experimentally manifest. That is in principle the R outcome of the state vector reduction or quantum jumps which seriously disturbed E. Schrödinger considerably [6]. Of course using the state vector collapse terminology is slightly inaccurate because it refers originally to the superposition which jumps into a single state at measurement but the connection is obvious [28,29]. In this way we have a mathematical explanation which is both causal, deterministic and realist for which we need not dwell on other aspects and physical theories such as the many world interpretation [7,8] or the black hole information problem [16,17].

Humans have an inbuilt intrinsic bias in their psyche against nothingness. Although we fear nothing like nothingness, being associated with death, we still do not regard it as physically real and integrate it on a fundamental level into the foundation of physics. So far we have been content with the zero introduced by the Indians and mediated to Europe by Arab mathematicians who saved our arithmetic and number system from the unbearable heaviness of Roman numbers.

The authors feel that the empty set of the Menger-Urysohn transfinite dimensional theory can do for physics what the zero did for mathematics when we extend the empty set dim $d_{nu} = -1$ to the totally empty set dim $d_{MU} = -\infty$. It may be instructive at this point to raise some fundamental philosophical problems which were considered in great depth around the middle of the last century in great depth. In that respect we may mention the views of M. Heidegger laid down in his book “Sein und Zeit” and later on the fundamental and famous work of J.P. Sartre’s, *Being and Nothingness*. In the somewhat flamboyant language of Sartre, he described the embedding of nothingness in being by liking it to a worm inside an apple. At the core of being nothingness is lurking. The exact mathematical formulation of the foundation of physics could similarly not be complete or consistent without including nothingness in the form of the empty set.

$$d_{MU} = -1$$

and

$$d_c^{(-1)} = \phi^2$$

and the totally empty set

$$d_{MU} = -\infty$$

and

$$d_c^{(-\infty)} = 0 \quad .$$

The distance between $d_{MU} = -1$ and $d_{MU} = -\infty$ is what is termed following a similar proposal by Mandelbrot the degree of emptiness of an empty set [29]. It is highly interesting to note that while too much philosophical questioning did not help build science, it did help separate between the empirical and testable from mere idle deep philosophical discussion. The situation started to improve with relativity and even more so with quantum mechanics. Nevertheless physics did not essentially change in essence with regard to the notion of nothingness. The introduction of the empty set on such a fundamental level clearly shows that our initial reaction to philosophy was misguided. Philosophy is part and parcel of real deep science and that is probably why fundamental science used to be called, at least in England, natural philosophy while at the time of G. Cantor it was not unusual to even employ the word metaphysics for the same meaning.

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