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Exceptional Lie groups hierarchy and some fundamental high energy physics equations

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Abstract

The exceptional Lie groups hierarchy E_8, E_7 and E_6 is extended to E_5 and E_4 and subsequently related to F_4 and G_2 in a fundamental equation using a super space $D^{(8)}$. The result is shown to represent the most likely expectation number for the elementary particles constituting an extended standard model.

1. Introductory remarks and a classification theorem for Lie groups

It is well-known that simple Lie algebra may be classified according to Dynkin to give the picture shown in Fig. 1. The notations An, Bn, Cn and Dn are all due to Cartan and correspond to SU($U+1$), SO($2n+1$), SP($2n$) and SO($2n$), respectively. In particular G_2, F_4, E_6, E_7 and E_8 are the exceptional Lie groups with order 14, 52, 78, 133 and 248, respectively [1–3]. Contemplating the telescopic structure of E_6, E_7, E_8, it is highly intriguing to note that

$$|E_6| + |E_7| + |E_8| = |\mathrm{SO}(\mathbf{32})| - \mathbf{37},$$

where $|\mathrm{SO}(\mathbf{32})| = |E_8 E_8| = \mathbf{496}$ are the massless gauge bosons of heterotic string theory [1–3].

It is the intention of the present short note to uncover further relevant and highly interesting relations of this hierarchical, telescopic type groups and relate it to the physics of high energy and unification [4–6].

2. An extension of the E_i exceptional Lie groups

Following Fig. 2, it is easily reasoned from the Dynkin diagrams that SO(10) and SU(5) maybe viewed as E_5 and E_4. Recalling that both are related to specific grand unification scenarios [5] while SU(5) is sub-group of SO(10) exactly as SO(10) is a sub-group of the E_i line, then we could extend the result of the preceding paragraph to

$$\sum_{i=4}^{i=8} |E_i| = |(\mathrm{SO}(\mathbf{32})| - \mathbf{37}) + |F_4| + |G_2| = K^{(32)},$$

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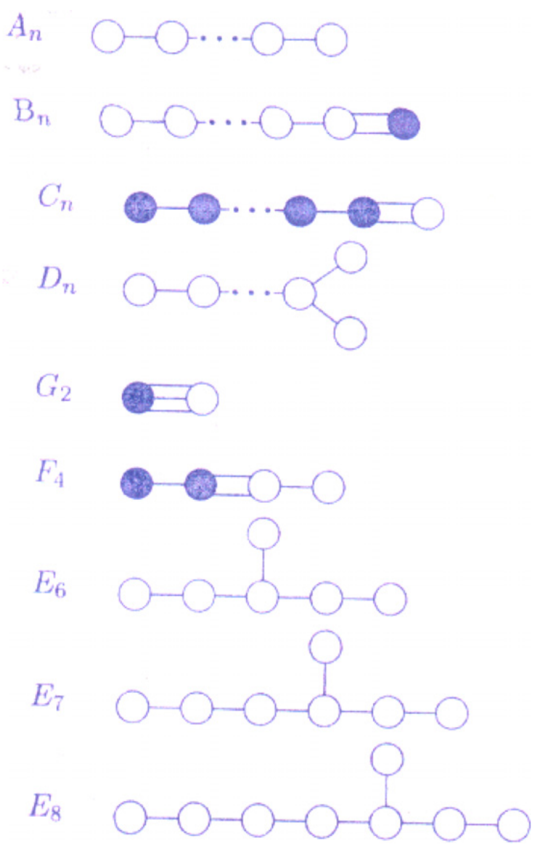

Fig. 1.

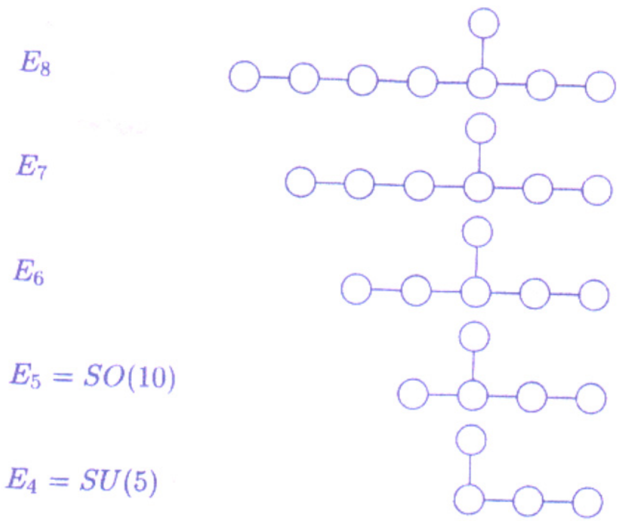

Fig. 2.

where $K^{(32)} = (32)(33)/2 = 528$ is the number of killing Vector fields for a maximally symmetric space [4] for $n = (8)(4) = 32$.

3. The F_4, G_2 and relation to P-Brane theory

From $P = 5$ Brane theory in $D = 11$ we know that the number of states is given by [4]

$$N^{11}(\text{Brane}) = \binom{11}{\mathbf{1}} + \binom{11}{\mathbf{2}} + \binom{11}{\mathbf{5}} = 11 + 55 + 462 = 528 = K^{(32)}.$$

In addition it is clear from the previous paragraphs that [1]

$$|G_2| + |F_4| = 14 + 52 = 66.$$

In other words

$$|G_2| + |F_4| = \text{number of string and membrane states} = \binom{11}{\mathbf{1}} + \binom{11}{\mathbf{2}} = 66.$$

Recalling that the likely number of elementary particles in an extended standard model was obtained from $K^{(23)}$ via a three-step symmetry breaking [4]

$$K^{(23)}/D^{(8)} = 528/8 = 66.$$

It is a trivial matter to see that

$$K^{(23)}/D^{(8)} = |G_2| + |F_4|.$$

4. A new fundamental equation for hierarchical exceptional Lie groups and conclusion

Recapitulating the results of the previous section, we see that the telescopic exceptional Lie groups hierarchy leads to

$$\sum_{i=4}^{i=8} |E_i|/D^{(8)} = |G_2| + |F_4| = N(\text{SM}) = 66.$$

This equation means simply that groups and in particular the Exceptional Lie groups hierarchy is at the very heart of the nature of high energy physical reality [2–7].

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